Discussion 2/23

2.1.4

2.1.6 (a)
(b) ✓

2.1.11. ✓

2.1.12.

2.2.4 ✓

2.2.7 -

2.2.8 ✓

2.2.9.

2.2.10.

2.2.11.

2.2.12.

2.2.18.

2.3.3. (a) ✓
(c) -

2.3.4 (d) ✓

2.3.7

2.3.15 (c) ✓

2.3.16

2.3.17

2.1.6 (b) Let $x_1 = 0$, $x_2 = 1$, $x_3 = -1$. Find the unique quadratic function, $f(x) = ax^2 + bx + c$ with sample vector $\vec{f} = (1, -2, 0)^T$.

$\vec{f}$ means $f(x_1) = 1$, $f(x_2) = -2$, $f(x_3) = 0$.

$\Rightarrow$ $f(0) = 1$, $f(1) = -2$, $f(-1) = 0$.

$a(0) + b(0) + c = 1 \Rightarrow c = 1$

$a(1) + b(1) + 1 = -2 \Rightarrow a + b = -3$

$a(1) + b(-1) + 1 = 0 \Rightarrow a - b = -1$

$2a = -4$

$\Rightarrow a = -2$

$b = -3 - a = -3 - (-2) = -1$

$f(x) = -2x^2 - x + 1$.

2.1.11 Prove that 1. $(-1)v = -v$ Already proved $0v = \vec{0}$.

2. $c\vec{0} = \vec{0}$

and 3. $cv = \vec{0} \Rightarrow c = 0$ or $v = \vec{0}$.

1. $v + (-1)v = 1v + (-1)v = (1 + (-1))v = 0v = \vec{0}$

So $(-1)v = -v$.

2. $c\vec{0} + c\vec{0} = c(\vec{0} + \vec{0}) = c\vec{0}$

$\Rightarrow c\vec{0} = \vec{0}$.

3. Assume $cv = \vec{0}$. If $c \neq 0$, then $cv = 0 \Rightarrow v = \frac{1}{c}\vec{0} = \vec{0}$.

Thus either $c = 0$ or $v = \vec{0}$.

2.2.4. Show that if $W \subseteq \mathbb{R}^3$ is a subspace containing the vectors $\underset{w_1}{\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}}, \underset{w_2}{\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}}, \underset{w_3}{\begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}}$, then $W = \mathbb{R}^3$.

Assume $W \subseteq \mathbb{R}^3$ is a subspace containing $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$.

To show $W = \mathbb{R}^3$, we need to show

$W \subseteq \mathbb{R}^3$ & $\mathbb{R}^3 \subseteq W$.
✓

Let $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$.

Want $a, b, c$ s.t. $aw_1 + bw_2 + cw_3 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & x \\ 2 & 0 & -1 & y \\ -1 & 1 & 3 & z \end{array}\right] \xrightarrow[R1+R3]{-2R1+R2} \left[\begin{array}{ccc|c} 1 & 2 & 0 & x \\ 0 & -4 & -1 & y-2x \\ 0 & 3 & 3 & x+z \end{array}\right]$$

$$\xrightarrow{3/4\,R2+R3} \left[\begin{array}{ccc|c} 1 & 2 & 0 & x \\ 0 & -4 & -1 & y-2x \\ 0 & 0 & 2\,1/4 & x+z+3/4\,y-3/2\,x \end{array}\right]$$

has soln.

So $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \text{span}\{w_1, w_2, w_3\}$

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in W$. So $\mathbb{R}^3 \subseteq W$. Thus $W = \mathbb{R}^3$.

2.2.8. Prove that the set of all solutions, $\vec{x}$, of a the linear system $A\vec{x} = \vec{b}$ forms a subspace iff the system is homogenous.

$\Rightarrow$ Assume it forms a subspace.

Then $\vec{0}$ is a soln.

So $A\vec{0} = \vec{b} \Rightarrow \vec{0} = \vec{b}$

$\Rightarrow$ homogenous.

$\Leftarrow$ Assume the system is homogenous, $A\vec{x} = \vec{0}$

Let $\vec{x}, \vec{y}$ be solns, $c \in \mathbb{R}$. Then $A\vec{x} = 0$ and $A\vec{y} = 0$.

Then $A(\vec{x} + \vec{y}) = A\vec{x} + A\vec{y} = \vec{0} + \vec{0} = \vec{0}$

So $\vec{x} + \vec{y}$ is a soln.

and $A(c\vec{x}) = cA\vec{x} = c\vec{0} = \vec{0}$

So $c\vec{x}$ is a soln.

Thus the set of solns forms a subspace.

2.3.3 (a) Is $\begin{pmatrix}1\\-2\\-3\end{pmatrix}$ in span $\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$ ?

i.e. do there exist $a, b \in \mathbb{R}$ s.t

$$a\begin{pmatrix}1\\1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}1\\-2\\-3\end{pmatrix}$$

$a = 1$
$a + b = -2$
$b = -3$

$a + b = 1 + -3 = -2$ ✓

Yes.

2.3.4 (d) Does $\left\{\begin{pmatrix}6\\-9\end{pmatrix}, \begin{pmatrix}-4\\6\end{pmatrix}\right\}$ span $\mathbb{R}^2$

$$\left[\begin{array}{cc|c} 6 & -4 & x \\ -9 & 6 & y \end{array}\right] \xrightarrow{3/2\,R1+R2} \left[\begin{array}{ccc} 6 & -4 & x \\ 0 & 0 & y + 3/2\,x \end{array}\right]$$

$\Rightarrow y = -3/2\,x$

Not soln for all $x, y$.

Does not span.

2.3.15 (c) Is $\begin{pmatrix}1-2x\\1-x\end{pmatrix}$ is span $\left\{\begin{pmatrix}1\\x\end{pmatrix}, \begin{pmatrix}x\\1\end{pmatrix}, \begin{pmatrix}x\\2x\end{pmatrix}\right\}$ ?

$$a\begin{pmatrix}1\\x\end{pmatrix} + b\begin{pmatrix}x\\1\end{pmatrix} + c\begin{pmatrix}x\\2x\end{pmatrix} = \begin{pmatrix}1-2x\\-1-x\end{pmatrix}$$

$\Rightarrow$ $a + bx + cx = 1 - 2x$ $\Rightarrow$ $a + (b+c)x = 1 - 2x$
$ax + b + 2cx = -1 - x$ $\qquad b + (a+2c)x = 1 - x$

$\Rightarrow$ $a = 1$ ; $b + c = -2$ $\Rightarrow$ $c = -2 + 1 = -1$
$-b = 1$ , $a + 2c = -1$ $\qquad 1 + 2(-1) = -1$ ✓ ; Yes.